

$$1. \boxed{V(x, y, z) = 125} \quad xy z \overset{\leftarrow}{=} 125$$

$$\boxed{f(x, y, z) = 2(xy + xz + yz)}$$

$$\boxed{g(x, y, z) = x \cdot y \cdot z - 125}$$

$$\boxed{xy z - 125 = 0}$$

$$g(x)$$

$$L(x, y, z, \lambda) = f(x) - \lambda g(x)$$

$$= 2xy + 2xz + 2yz - \lambda(xy z - 125)$$

$$\bullet \frac{\partial L}{\partial x} = 2y + 2z - \lambda y z = 0 \quad \bullet \frac{\partial L}{\partial z} = 2x + 2y - \lambda xy = 0$$

$$\bullet \frac{\partial L}{\partial y} = 2x + 2z - \lambda x z = 0 \quad \bullet \frac{\partial L}{\partial \lambda} = -xy z + 125 \rightarrow xy z = 125$$

$$xy z = 125$$

$$2x + 2z - \lambda x z = 0 \quad 2x + 2y - \lambda xy = 0$$

$$2x + 2z = \lambda x z \quad 2x + 2y = \lambda xy$$

$$\frac{2x + 2z}{xz} = \lambda \quad \frac{2x + 2y}{xy} = \lambda$$

$$\frac{2x + 2z}{xz} = \frac{2x + 2y}{xy} \rightarrow \frac{2x + 2z}{2x + 2y} = \frac{yz}{xy}$$

$$\frac{x + z}{xz} = \frac{x + y}{xy}$$

$$\frac{x + z}{x + y} = \frac{z}{y}$$

$$(x + z)(xy) = (x + y)(xz)$$

$$x^2 y + x y z = x^2 z + x y z \rightarrow$$

$$x^2 y = x^2 z$$

$$\boxed{y = z}$$

$$(x + z)(y) = (x + y)(z)$$

$$xy + yz = xz + yz$$

$$xy + yz = xz + yz$$

$$x = y = z$$

$$xyz = 125$$

$$x \cdot x \cdot x = 125$$

$$x^3 = 125$$

$$x = y = z$$

$$y = z$$

$$x = 5$$

$$x = 5$$

$$5 \times 5 \times 5$$

$$\boxed{x + 2y = 2} \rightarrow \boxed{g(x, y) = x + 2y - 2}$$

$$f(x, y) = x \cdot y$$

$$\begin{aligned} \mathcal{L}(x, y, \lambda) &= f(x, y) - \lambda g(x, y) \\ &= xy - \lambda (x^2 + y^2 - 2) \end{aligned}$$

$$\bullet \frac{\partial \mathcal{L}}{\partial x} = y - \lambda = 0 \quad , \quad \frac{\partial \mathcal{L}}{\partial \lambda} = -x - 2y + 2 = 0 \quad , \quad$$

$$\bullet \frac{\partial \mathcal{L}}{\partial y} = x - 2\lambda = 0$$

$$\begin{cases} \lambda = y \\ 2\lambda = x \end{cases}$$

$$-2\lambda - 2\lambda + 2 = 0$$

$$-4\lambda + 2 = 0$$

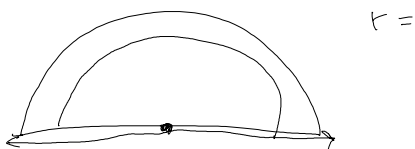
$$-4\lambda = -2$$

$$\lambda = \frac{1}{2}$$

$$y = \frac{1}{2} \quad x = 1$$

$$f(x, y) = 1 \cdot \frac{1}{2} = \frac{1}{2} //$$

Parameter $\boxed{0 \leq r \leq 1} \rightarrow -1 \leq x \leq 1$
 $0 \leq \theta \leq \pi$ ~~and~~ $0 \leq y \leq \sqrt{1-x^2}$



$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$\begin{aligned} y^2 &= 1-x^2 \\ y &= \sqrt{1-x^2} \end{aligned}$$

3) $f(r, \theta) = r^2 \sin \theta \cos \theta$

$$x = r \cos \theta$$

$$y = r \sin \theta \quad f(x, y) = x \cdot y$$

$$\int_0^\pi \int_0^1 r^3 \sin \theta \cos \theta \, dr \, d\theta$$

$$0 \leq r \leq 1 \rightarrow -1 \leq x \leq 1$$

$$0 \leq \theta \leq \pi \rightarrow 0 \leq y \leq \sqrt{1-x^2}$$

$$r^3 \sin \theta \cos \theta = x \cdot y \sqrt{x^2 + y^2}$$

$\downarrow$ $r \sin \theta = y$ $r \cos \theta = x$

?? $x^2 + y^2 = r^2$

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} x \cdot y \cdot \sqrt{x^2 + y^2} \, dy \, dx$$

$$\int_{-1}^1 x \left(\int_0^{\sqrt{1-x^2}} y \sqrt{x^2 + y^2} \, dy \right) dx$$

... $\int_{-1}^1 \dots \sqrt{1-x^2} \, dx$

$$I_y = \int_0^{\sqrt{1-x^2}} y \sqrt{x^2+y^2} dy$$

$$u = \sqrt{x^2+y^2}$$

$$u^2 = x^2 + y^2$$

$$y=0 \rightarrow u = \sqrt{x^2} = |x|$$

$$y = \sqrt{1-x^2} \rightarrow u = \sqrt{x^2 + 1 - x^2}$$

$$u = 1$$

$$\frac{2u}{2} du = \frac{2y}{2} dy$$

$$u du = y dy$$

$$I_y = \int_{|x|}^1 u \cdot u du = \int_{|x|}^1 u^2 du$$

$$= \frac{u^3}{3} \Big|_{|x|}^1 = \frac{1}{3} - \frac{|x|^3}{3}$$

$$\int_{-1}^1 x \cdot \frac{1 - |x|^3}{3} dx$$

$$= \frac{1 - |x|^3}{3} \Big|_{-1}^1 \quad I_y$$

$$\frac{1}{3} \int_{-1}^1 x - |x|^3 \cdot x dx$$

$$x, x \geq 0$$

$$\int_{-1}^0 \& \int_0^1$$

$$-x, x < 0$$

$$\frac{1}{3} \left[\int_{-1}^0 x - (-x)^3 \cdot x \, dx + \int_0^1 x - x^3 \cdot x \, dx \right]$$

$$\left[\int_{-1}^0 x + x^4 \, dx + \int_0^1 x - x^4 \, dx \right]$$

$$\frac{x^2}{2} \Big|_{-1}^0 + \frac{x^5}{5} \Big|_{-1}^0 + \frac{x^2}{2} \Big|_0^1 - \frac{x^5}{5} \Big|_0^1$$

$$0 - \left(\frac{1}{2}\right) + 0 - \left(-\frac{1}{5}\right) + \frac{1}{2} - 0 - \left(\frac{1}{5} - 0\right)$$

$$-\frac{1}{2} + \frac{1}{5} + \frac{1}{2} - \frac{1}{5}$$

$$0 \cdot \frac{1}{3} = 0 //$$

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} x \cdot y \cdot \sqrt{x^2+y^2} \, dy \, dx$$

$$\int_{-1}^1 x \left[\int_0^{\sqrt{1-x^2}} y \sqrt{x^2+y^2} \, dy \right] dx$$

$$u = \sqrt{x^2+y^2}$$

$$y=0 \rightarrow u = \sqrt{x^2+(0)^2} = \sqrt{x^2} = |x|$$

$$\sqrt{1-x^2} = \sqrt{1-1} = 0$$

$$u = \sqrt{x^2 + y^2}$$

$$u^2 = x^2 + y^2$$

$$2u \, du = 2y \, dy$$

$$u \, du = y \, dy$$

$$\frac{u}{y} \, du = dy$$

$$y=0 \rightarrow u = \sqrt{x^2 + (0)^2} = \sqrt{x^2} = |x|$$

$$y = \sqrt{1-x^2} \rightarrow u = \sqrt{x^2 + (\sqrt{1-x^2})^2} = \sqrt{x^2 + 1 - x^2} = \sqrt{1} = 1$$

$$|x| \leq u \leq 1$$

$$\int_{|x|}^1 \cancel{u} \cdot \cancel{y} \cdot \frac{\cancel{u}}{\cancel{y}} \, du = \int_{|x|}^1 u^2 \, du$$